Lösen Sie die folgenden Integrale durch Substitution:

a) $\int \sin(3x)dx$

Lösung:

$$u = 3x; \ \frac{du}{dx} = 3; \ dx = \frac{du}{3}$$

$$\int \sin(3x)dx = \frac{1}{3}\int \sin(u) \cdot du = -\frac{1}{3}\cos(u) + C = -\frac{1}{3}\cos(3x) + C$$

b) $\int e^{2x+5}dx$

Lösung:

$$u = 2x + 5; \ \frac{du}{dx} = 2; \ dx = \frac{du}{2}$$

$$\frac{1}{2}\int e^u dx = \frac{1}{2}e^u + C = \frac{1}{2}e^{2x+5} + C$$

c) $\int xe^{-x^2}dx$

Lösung:

$$u = -x^2; \ \frac{du}{dx} = -2x; \ dx = \frac{du}{-2x}$$

$$\int xe^{-x^2}dx = \int xe^u \frac{du}{-2x} = -\frac{1}{2}\int e^u \cdot du = -\frac{1}{2}e^u + C = -\frac{1}{2}e^{-x^2} + C$$

d) $\int \frac{dx}{3x-2}$

Lösung:

$$u = 3x - 2; \ \frac{du}{dx} = 3; \ dx = \frac{du}{3}$$

$$\int \frac{dx}{3x-2} = \int \frac{1}{u} \cdot \frac{du}{3} = \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln|3x-2| + C$$

e) $\int \frac{\cos(x)}{1+\sin^2(x)}dx$

Lösung:

$u=\sin(x);\ \frac{du}{dx}=\cos(x);\ dx=\frac{du}{\cos(x)}$

$\int \frac{\cos(x)}{1+\sin^2(x)}dx=\int \frac{\cos(x)}{1+u^2}\frac{du}{\cos(x)}=\int \frac{1}{1+u^2}du=\arctan(u)+C=\arctan(\sin(x))+C$

f) $\int \frac{\arctan(x)}{1+x^2}dx$

Lösung:

$u=\arctan(x);\ \frac{du}{dx}=\frac{1}{1+x^2};\ dx=du(1+x^2)$

$\int \frac{\arctan(x)}{1+x^2}dx=\int \frac{u}{1+x^2}du(1+x^2)=\int u\cdot du=\frac{u^2}{2}+C=\frac{1}{2}\arctan^2(x)+C$

g) $\int \sin^3(x)dx \qquad Anmerkung:\ \sin^2(x)+\cos^2(x)=1$

Lösung:

$\int \sin^3(x)dx=\int \sin(x)(1-\cos^2(x))dx; \qquad u=\cos(x);\ \frac{du}{dx}=-\sin(x);\ dx=\frac{du}{-\sin(x)}$

$=\int \sin(x)(1-u^2)\frac{du}{-\sin(x)}=-\int (1-u^2)\cdot du=-u+\frac{u^3}{3}+C=-\cos(x)+\frac{1}{3}\cos^3(x)+C$

h) $\int \frac{\sin^5(x)}{\cos^7(x)}dx \qquad Anmerkung:\ \frac{\sin(x)}{\cos(x)}=\tan(x)$

Lösung:

$\int \frac{\sin^5(x)}{\cos^7(x)}dx=\int \frac{\tan^5(x)}{\cos^2(x)}dx \qquad u=\tan(x);\ \frac{du}{dx}=\frac{1}{\cos^2(x)};\ dx=du\cdot\cos^2(x)$

$=\int \frac{u^5(x)}{\cos^2(x)}du\cdot\cos^2(x)=\int u^5du=\frac{1}{6}u^6+C=\frac{1}{6}\tan^6(x)+C$

i) $\int \frac{e^x}{e^x + a} dx$

Lösung:

$$u = e^x + a; \ \frac{du}{dx} = e^x; \ dx = \frac{du}{e^x}$$

$$\int \frac{e^x}{e^x + a} dx = \int \frac{e^x}{u} \cdot \frac{du}{e^x} = \int \frac{1}{u} du = \ln|u| + C = \ln|e^x + a| + C$$

j) $\int \cot(x) dx$ $\qquad$ $Anmerkung: \ \frac{\cos(x)}{\sin(x)} = \cot(x)$

Lösung:

$$u = \sin(x); \ \frac{du}{dx} = \cos(x); \ dx = \frac{du}{\cos(x)}$$

$$= \int \frac{\cos(x)}{u} \cdot \frac{du}{\cos(x)} = \int \frac{1}{u} du = \ln|u| + C = \ln|\sin(x)| + C$$

k) $\int x^5 \sqrt{x^3 - 3} dx$

Lösung:

$$u = x^3 - 3 \qquad x^3 = u + 3 \qquad \frac{du}{dx} = 3x^2 \qquad dx = \frac{du}{3x^2}$$

$$= \int x^5 \sqrt{u} \frac{du}{3x^2} = \int x^3 x^2 \sqrt{u} \frac{du}{3x^2} = \int (u+3) u^{\frac{1}{2}} \frac{du}{3} = \frac{1}{3} \int \left(u^{\frac{3}{2}} + 3u^{\frac{1}{2}} \right) du =$$

$$\frac{1}{3} \left(\frac{2}{5} u^{\frac{5}{2}} + \frac{3 \cdot 2}{3} u^{\frac{3}{2}} \right) + C = \frac{2}{15} u^{\frac{5}{2}} + \frac{2}{3} u^{\frac{3}{2}} + C$$

$$= \underline{\underline{\frac{2}{15} (x^3 - 3)^{\frac{5}{2}} + \frac{2}{3} (x^3 - 3)^{\frac{3}{2}} + C}}$$