6.1a) $\vec{a}^\circ = \dfrac{\vec{a}}{|\vec{a}|}$; $|\vec{a}| = \sqrt{2^2+1^2} = \sqrt{5}$

$$\vec{a}^\circ = \underline{\underline{\frac{1}{\sqrt{5}}\begin{pmatrix}2\\1\\0\end{pmatrix}}} = \underline{\underline{\begin{pmatrix}0{,}8944\\0{,}4472\\0\end{pmatrix}}}$$

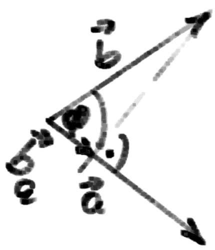
b) $\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cdot\cos\varphi \Rightarrow \cos\varphi = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$

$|\vec{b}| = \sqrt{5} = |\vec{a}|$ hier; also: $\cos\varphi = \dfrac{\vec{a}\cdot\vec{b}}{(\sqrt{5})^2}$

$\vec{a}\cdot\vec{b} = 2\cdot 0 + 1\cdot 1 + 0\cdot 2 = 1$

$\cos\varphi = \dfrac{1}{5}$, also $\underline{\underline{\varphi = 1{,}37 \text{ rad } (\hat{=}\, 78{,}4^\circ)}}$

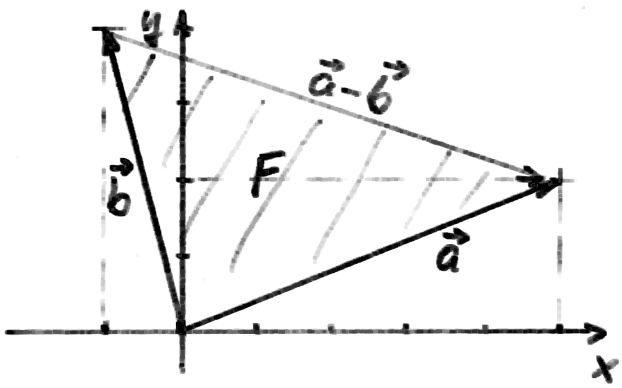
c)



$|\vec{b}_a| = |\vec{b}|\cdot\cos(\varphi) = \sqrt{5}\cdot\dfrac{1}{5} = \dfrac{1}{\sqrt{5}}$

$$\vec{b}_a = |\vec{b}_a|\cdot\vec{a}^\circ = \frac{1}{\sqrt{5}}\cdot\frac{1}{\sqrt{5}}\begin{pmatrix}2\\1\\0\end{pmatrix} = \underline{\underline{\begin{pmatrix}0{,}4\\0{,}2\\0\end{pmatrix}}}$$

6.2



$$F = \frac{1}{2}\cdot(\vec{a}\times\vec{b}) = \frac{1}{2}(a_1 b_2 - a_2 b_1) = \frac{1}{2}\begin{vmatrix}a_1 & a_2\\ b_1 & b_2\end{vmatrix}$$

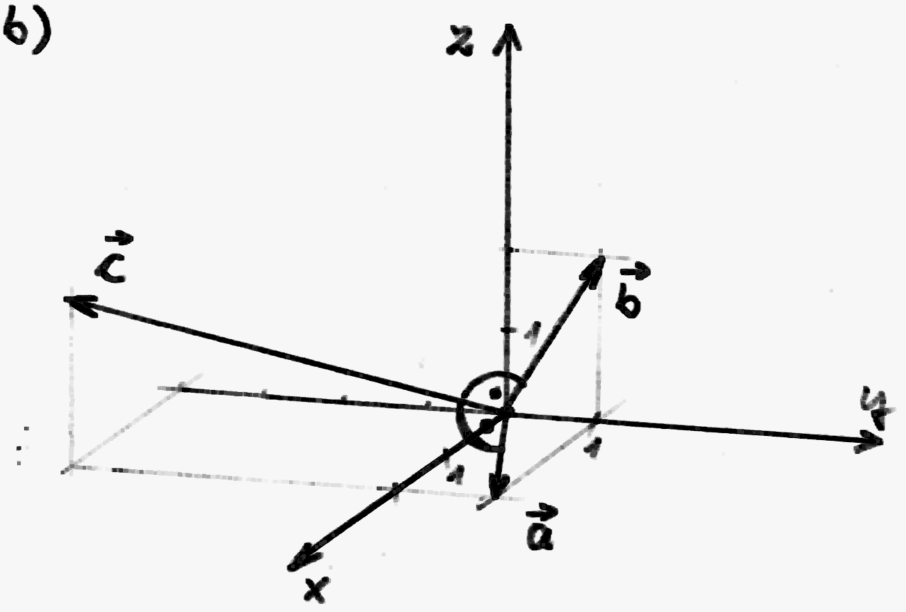
$$F = \frac{1}{2}\left(5\cdot 4 - (-1)\cdot 2\right) m^2 \underline{\underline{= 11\, m^2}}$$

Hinweis: Das Kreuzprodukt $\vec{b}\times\vec{a}$ ergibt hier eine negative Fläche.

6.3)

a) $\vec{a} \times \vec{b} = \begin{pmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 2 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 2 \end{pmatrix}$

b)



6.4) a) $\vec{a}, \vec{b}, \vec{g}$ sind linear unabhängig, wenn

$$D = \begin{vmatrix} a_1 & b_1 & g_1 \\ a_2 & b_2 & g_2 \\ a_3 & b_3 & g_3 \end{vmatrix} \neq 0 \text{ gilt.}$$

$$D = \begin{vmatrix} 2 & 0 & -2 \\ 1 & 1 & 0 \\ 0 & 2 & 1 \end{vmatrix} = 2\cdot(1) + 0 - 2\cdot(2) = \underline{\underline{-2 \neq 0}}$$

b) Weil $\vec{a}, \vec{b}, \vec{g}$ linear unabhängig sind, kann man $\vec{g}$ nicht als Linearkombination von $\vec{a}, \vec{b}$ angeben.

$\vec{a}, \vec{b}, \vec{g}$ sind nicht komplanar!