Lösungen zu Mathe 1 (B) – Blatt 8 / Wiebe

zu 8.1) b) $r = \sqrt{Re^2(\underline{z}) + Im^2(\underline{z})}$

$r_1 = \sqrt{2^2+1^2} = \sqrt{5}$; $r_2 = \sqrt{2^2+2^2} = 2\cdot\sqrt{2}$

$\varphi = \arctan\left(\frac{Im(\underline{z})}{Re(\underline{z})}\right) = \arcsin\left(\frac{Im(\underline{z})}{r}\right) = \ldots$

$\varphi_1 = \arctan\left(\frac{1}{2}\right) = 0{,}464\ \text{rad} \triangleq 26{,}6°$

(1. Quadrant)

$\varphi_2 = \arctan\left(\frac{2}{-2}\right) + \pi = 2{,}356\ \text{rad} \triangleq 135°$

(2. Quadrant)

Damit:

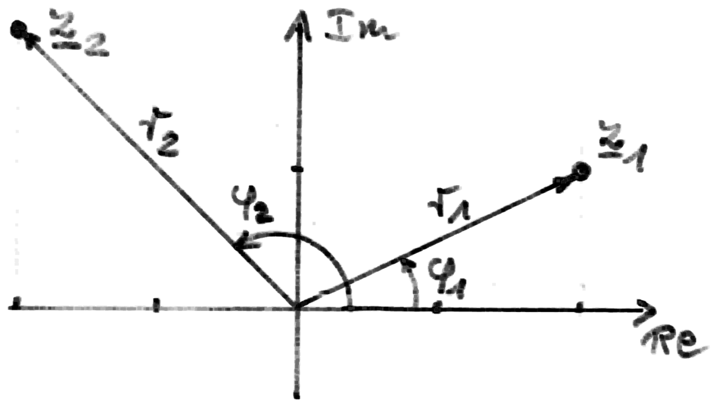
$\underline{z}_1 = \sqrt{5}\cdot(\cos(0{,}464\ \text{rad}) + i\cdot\sin(0{,}464\ \text{rad}))$

$= \sqrt{5}\cdot e^{i\cdot 0{,}464\ \text{rad}}$

$\underline{z}_2 = 2\cdot\sqrt{2}\cdot(\cos(2{,}356\ \text{rad}) + i\cdot\sin(2{,}356\ \text{rad}))$

$= 2\cdot\sqrt{2}\cdot e^{i\cdot 2{,}356\ \text{rad}}$

a)



c) $\underline{z} = (2-2) + i\cdot(1+2) = 0 + i\cdot 3$

d) $\underline{z} = \sqrt{5}\cdot 2\cdot\sqrt{2}\cdot e^{i\cdot(0{,}464+2{,}356)\ \text{rad}} = 6{,}325\cdot e^{i\cdot 2{,}82\ \text{rad}}$

$\underline{z} = (2+i\cdot 1)\cdot(-2+i\cdot 2) = 2\cdot(-2) - 1\cdot 2 + i\cdot(2\cdot 2 + 1\cdot(-2))$

$\underline{z} = -6 + i\cdot 2 = 6{,}325\cdot(\cos(2{,}82\ \text{rad}) + i\cdot\sin(2{,}82\ \text{rad}))$

— Probe —

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zu 8.1)

e) $$\underline{z} = \frac{\sqrt{5} \cdot e^{i \cdot 0{,}464\,\mathrm{rad}}}{2\sqrt{2} \cdot e^{i \cdot 2{,}356\,\mathrm{rad}}} = 0{,}79 \cdot e^{i \cdot (-1{,}892\,\mathrm{rad})}$$

oder

$$\underline{z} = \frac{2 + i \cdot 1}{-2 + i \cdot 2} = \frac{(2+i\cdot 1)(-2-i\cdot 2)}{(-2+i\cdot 2)(-2-i\cdot 2)} = \frac{-4+2+i(-4-2)}{(-2)^2 - (i\cdot 2)^2}$$

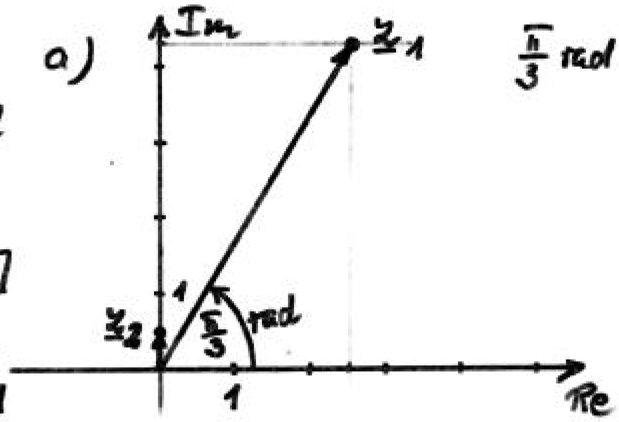
$(a^2 - b^2)$

$$\underline{\underline{z}} = \frac{-2 - i\cdot 6}{4 + 4} = \underline{\underline{-\frac{1}{4} - i \cdot \frac{3}{4}}}$$

$$\underline{z} = 0{,}79 \cdot (\cos(-1{,}892\,\mathrm{rad}) + i \cdot \sin(-1{,}892\,\mathrm{rad}))$$ Probe!

zu 8.2)

a)



$\frac{\pi}{3}\,\mathrm{rad} \mathrel{\hat{=}} 60°$

b) $$\underline{z} = 5 \cdot 0{,}5 \cdot e^{i\left(\frac{\pi}{3} + \frac{\pi}{2}\right)\mathrm{rad}}$$

$$\underline{z} = 2{,}5 \cdot e^{i \cdot \frac{5}{6}\pi\,\mathrm{rad}}$$

[$\hat{=}$ 150°]

c) $$\underline{z}_3 = \frac{1}{i \cdot 0{,}5} = 2 \cdot e^{-i\frac{\pi}{2}\mathrm{rad}}$$

$$\underline{\underline{z}_3 = -i \cdot 2}$$

$$\underline{z}_1 \cdot \underline{z}_3 = 5 \cdot 2 \cdot e^{i\left(\frac{\pi}{3} - \frac{\pi}{2}\right)\mathrm{rad}} = 10 \cdot e^{-i\frac{\pi}{6}\mathrm{rad}}$$

[$\hat{=}$ -30°]

d) Umwandlung von $\underline{z}_1$:

$$\underline{z}_1 = 5 \cdot \left(\cos\left(\tfrac{\pi}{3}\,\mathrm{rad}\right) + i \cdot \sin\left(\tfrac{\pi}{3}\,\mathrm{rad}\right)\right) = 2{,}5 + i \cdot 4{,}33$$

Summe: $$\underline{z} = 2{,}5 + 0 + i \cdot (4{,}33 + 0{,}5)$$

$$\underline{\underline{\underline{z} = 2{,}5 + i \cdot 4{,}83}}$$